

COMPARISON OF EQUATIONS FOR THE FDTD SOLUTION IN ANISOTROPIC AND DISPERSIVE MEDIA *

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The FDTD solution procedure developed by K. S. Yee has in recent years been extended to dispersive and anisotropic media to handle materials such as magnetized ferrites and plasmas. The constitutive relations for the dispersive material can be introduced into the solution through either a convolution integral or a differential equation. The convolution form, using a recursive update procedure was developed by R. J. Luebbers, F. P. Hunsberger and K. S. Kunz (IEEE Trans. Electromagn. Compat., Aug. 1990.) In this recursive convolution (RC) method the susceptibility function of the material is represented as a sum of exponential functions of time, which describes many common dielectric and magnetic materials. The convolution integral relating \mathbf{B} to \mathbf{H} or \mathbf{D} to \mathbf{E} can then be updated by a simple recursion relation as part of the FDTD update procedure. The treatment of materials that are both anisotropic and dispersive was developed by F. P. Hunsberger (Ph. D. Thesis, Penn. State Univ., 1991) and the solution in 3D was demonstrated by J. Schuster and R. Luebbers (Proc. of the 12th Annual Rev. of Comp. Electromag., 1996.) For anisotropic material the susceptibility functions become tensors that couple the field components and require averaging to obtain components that are not directly available in the Yee formulation.

The development of the RC solution can lead to several different forms of the update equations. The time derivative of the convolution integral can involve the derivative of the field, or integrating by parts can put the derivative on the susceptibility function. Reduction of these two results to discrete form leads to slightly different update equations. Also, the choice of the evaluation time and integration limit of the convolution integral can lead to differences in the discrete update equation. These different forms of the solution are compared for accuracy and stability for time increments approaching the Courant limit. It is found that slightly greater accuracy and greater stability are obtained with the convolution evaluated at the time of the equation, a half step before the field being evaluated, using a pulse approximation of the integral ending in a half pulse. Modifications of this result lead to somewhat simpler but less stable equations. In the case that the susceptibility function starts at zero for time equal to zero the equations for anisotropic and dispersive material simplify greatly, requiring only the addition of the RC term to the normal FDTD equations, without further coupling of the field components. While 3D solutions are considered here, the accuracy and stability are demonstrated for the 1D problem of normal incidence on a slab of ferrite or plasma with biasing field in the direction of propagation, since simple analytic solutions are available for this problem.

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